Rigorous Analysis of Open Microstrip Lines of Arbitrary Cross Section in Bound and Leaky Regimes

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Abstract —**The problem of an open microstrip line of arbitrary cross section is solved by an integral equation technique in conjunction with the method of moments. The approach is general and can handle as special cases multiple strips and strips of finite or infinitesimal thickness. It applies to both the fundamental and higher order modes, whether in the bound or the leaky regime. Computed dispersion curves and modal current distributions are presented for several cases of interest and, where possible, are compared with published data.**

I. Introduction

Although open microstrip lines have been analyzed by both spectral-domain [1], [2] and space-domain [3], [4] integral equation methods, these analyses are not easily extendable to lines whose upper conductor is not an infinitesimally thick, planar strip. The authors are only aware of two publications where more general cross sections, namely rectangular [5] and circular [6] are considered. There is also a scarcity of results for higher order modes in the leaky regime [7].

In this paper, we present a rigorous dispersion analysis of open microstrip lines of arbitrary cross-sectional profile based on the mixed-potential electric field integral equation (MPIE) [8], [9]. We prefer the MPIE to several other possible forms of the electric field integral equation (EFIE), because it requires only *potential forms* of the Green's function, which are less singular and converge faster than the *field forms* needed in other EFIE's. Another important advantage of the MPIE is its conformity to well-established numerical solution techniques, originally developed for objects in free space [10].

II. Formulation

Consider a transmission line formed by an infinite, perfect conductor above a grounded dielectric slab of relative permittivity ϵ_r, as illustrated in Fig. 1. The cross-sectional profile L of the conductor may be arbitrary, but —as indicated in Fig. 1—the solution procedure requires

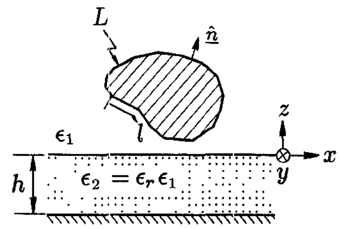

Fig. 1. PEC cylinder of arbitrary cross section above a grounded dielectric slab.

that it be approximated by straight line segments. An $e^{j\omega t}$ time dependence is assumed and suppressed. Since the structure is of infinite extent and uniform along the y axis, we postulate that the associated fields, as well as the current density $\boldsymbol{J}$ on L, vary with y as $e^{-jk_y y}$, where k_y is the propagation constant, which may be complex. Hence, we set $k_y/k_0 = \beta - j\alpha$, where β and α are the phase and attenuation (leakage) constants, respectively, normalized to the free-space wavenumber k_0. In the absence of external excitation, the objective is to compute the propagation constants and the associated currents for the fundamental mode and for the first few higher order modes that can be supported by the transmission line.

As was already mentioned, our approach is to formulate an MPIE and to solve it numerically by the method of moments [10]. We obtain the desired MPIE by specializing to the present two-dimensional case one of the three general MPIE's developed recently by the authors [11]. The result is

$$\hat{\boldsymbol{n}} \times \left\{ j\omega \boldsymbol{A}(l) + (\nabla_t - jk_y \hat{\boldsymbol{y}})\Phi(l) \right\} = 0, \qquad l \in L \quad (1)$$

where $\hat{\boldsymbol{n}}$ is a unit vector normal to L (cf. Fig. 1), ∇_t is the transverse operator nabla, $\boldsymbol{A}$ is a modified vector potential [8], which is related to $\boldsymbol{J}$ as

$$\boldsymbol{A}(l) = \int_L \underline{\underline{\boldsymbol{K}}}^A(l|l') \cdot \boldsymbol{J}(l')\, dl' \qquad (2)$$

and Φ is a scalar potential, which is related to the charge density q as

$$\Phi(l) = \int_L K^{\Phi}(l|l') q(l')\, dl' \qquad (3)$$

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where, in view of the continuity equation,

$$q(l)=\frac{j}{\omega}(\nabla_t - jk_y\hat{y})\cdot \boldsymbol{J}(l). \tag{4}$$

We say that (1) is in a mixed-potential form, because it involves both the vector and the scalar potential, the former expressed in terms of $\boldsymbol{J}$, and the latter in terms of q.

The expressions for the dyadic kernel $\underline{\underline{K}}^A$ and the scalar kernel K^{Φ} comprise improper spectral integrals of the form

$$S[f(k_x)]=\int_{-\infty}^{\infty} f(k_x)e^{-jk_{z1}(z+z')}e^{-jk_x(x-x')}\,dk_x \tag{5}$$

where k_x is the Fourier domain counterpart of x. With this notation, the nonzero elements of $\underline{\underline{K}}^A$ and K^{Φ} can be expressed as [12]

$$K^A_{xx}=K^A_{yy}=\frac{\mu_0}{4\pi}\left\{g(|\boldsymbol{r}-\boldsymbol{r}'|)-S\left[\frac{k_{z2}\cot(k_{z2}h)-jk_{z1}}{jk_{z1}D^h}\right]\right\} \tag{6}$$

$$K^A_{xz}=-K^A_{zx}=\frac{\mu_0}{4\pi}(\epsilon_r-1)S\left[\frac{2jk_x}{D^eD^h}\right] \tag{7}$$

$$K^A_{yz}=-K^A_{zy}=\frac{\mu_0}{4\pi}(\epsilon_r-1)S\left[\frac{2jk_y}{D^eD^h}\right] \tag{8}$$

$$K^A_{zz}=\frac{\mu_0}{4\pi}\left\{g(|\boldsymbol{r}-\boldsymbol{r}'|)+S\left[\frac{j\epsilon_r k_{z1}+k_{z2}\tan(k_{z2}h)}{jk_{z1}D^e}+(\epsilon_r-1)\frac{2jk_{z1}}{D^eD^h}\right]\right\} \tag{9}$$

$$K^{\Phi}=\frac{1}{4\pi\epsilon_0}\left\{g(|\boldsymbol{r}-\boldsymbol{r}'|)-S\left[\frac{k_{z2}\cot(k_{z2}h)-jk_{z1}}{jk_{z1}D^h}+(\epsilon_r-1)\frac{2jk_{z1}}{D^eD^h}\right]\right\} \tag{10}$$

where

$$D^e=j\epsilon_r k_{z1}-k_{z2}\tan(k_{z2}h) \tag{11}$$

$$D^h=jk_{z1}+k_{z2}\cot(k_{z2}h) \tag{12}$$

$$k_{zi}=\sqrt{\kappa_i^2-k_x^2}, \qquad i=1,2 \tag{13}$$

$$\kappa_i=\sqrt{k_i^2-k_y^2}, \qquad i=1,2 \tag{14}$$

and

$$g(|\boldsymbol{r}-\boldsymbol{r}'|)=\frac{\pi}{j}H_0^{(2)}(\kappa_1|\boldsymbol{r}-\boldsymbol{r}'|). \tag{15}$$

Here, $H_0^{(2)}$ is the zero-order Hankel function of the second kind, $k_1=\omega\sqrt{\mu_0\epsilon_0}$ and $k_2=\omega\sqrt{\mu_0\epsilon_0\epsilon_r}$ are the wavenumbers of the upper medium (which is taken to be free space) and the slab, respectively. The vectors $\boldsymbol{r}$ and $\boldsymbol{r}'$ refer, respectively, to the field and source points in the xz plane and are on L uniquely specified by l and l'.

As was already alluded to, the above equations represent just one of the three MPIE's developed in [11]. That more than one MPIE is possible is a direct consequence of the nonuniqueness of the vector potential in the presence of a dielectric interface. This point can be better explained as follows. Referring to Fig. 1, consider an x-directed Hertzian dipole above the slab. As is well known [13], two components of the vector potential $\boldsymbol{A}$ are needed in this case to satisfy the boundary conditions at the interface. In an early paper, Hoerschelmann [14] used A_z in addition to the primary A_x component, and this became the preferred choice in the literature. However, as was more recently pointed out [15], one might use as well A_x and A_y (or even A_y and A_z). Following [11], we characterize the former choice (A_x, A_z) as traditional, and the latter (A_x, A_y) as alternative.

There is also a difficulty associated with the scalar potential Φ. Namely, as first discussed by Mosig and Gardiol [16], the scalar kernel K^{Φ} may be interpreted as the potential of a single point charge associated with a Hertzian dipole. However, it was recently shown by Michalski [17] that—unless the alternative form of the vector potential is employed and the objective is confined to a single dielectric layer—the scalar potentials associated with the horizontal and vertical dipoles are different. This poses a dilemma, since only one K^{Φ} can appear in the MPIE. As first suggested by Michalski [8], this difficulty can be remedied by choosing K^{Φ} to be the scalar potential of either the horizontal *or* the vertical dipole and by properly modifying the vector potential kernel $\underline{\underline{K}}^A$. Hence, choosing K^{Φ} to be the scalar potential associated with a horizontal dipole in conjunction with the alternative or traditional vector potentials leads to two different MPIE's, referred to in [11] as formulations A and C, respectively. On the other hand, choosing K^{Φ} to be the scalar potential of a vertical dipole in conjunction with either the traditional or alternative vector potential leads to a third MPIE, which is referred to in [11] as formulation B. Depending on the geometry of the problem, one of the formulations may be preferable to the others. The reader is referred to [11] for a detailed discussion.

The MPIE listed above corresponds to formulation C of [11] (traditional vector potential in conjunction with a scalar potential of a horizontal dipole), which is particularly well suited to the analysis of planar microstrips, in view of the fact that $K^A_{xx}=K^A_{yy}$ and $K^A_{xy}=K^A_{yx}=0$. Consequently, only one spectral integral arises in the computation of $\boldsymbol{A}$.

III. Solution Procedure

Since (1) has a mixed-potential form, it is amenable to the moment method procedures of Glisson and Wilton [10] Although the latter were originally developed for objects in free space, they can be readily adapted to the present case. We note that except for the presence of the spectral integrals, the only major difference between (1) and its free-space counterpart is the dyadic character of the vector potential kernel, which must be properly accounted for in the solution procedure. Apart from that, the solution of (1) proceeds in a standard way [10]. Hence, we employ piecewise-constant (pulse) and piecewise-linear (triangle) basis functions to represent, respectively, the longitudinal and transverse components of $\boldsymbol{J}$. The same functions are used to "test" the equations in longitudinal and transverse directions; however, the testing integrals are approximated by a one-point rectangular rule quadrature. For more details, the reader is referred to [10] and [18].

As a result of the procedure summarized above, a homogeneous system of simultaneous algebraic equations is obtained for the current expansion coefficients, which has nontrivial solutions for those values of k_y that cause its determinant to vanish. Hence, to obtain the propagation constants of the various modes of the microstrip, a search is performed, using Müller's method [19], for the zeros of the determinant in the complex k_y plane. For each propagation constant, the homogeneous system is solved for the corresponding modal current distribution.

The spectral integrals (5) must be repeatedly evaluated by numerical quadrature along suitable paths in the complex k_x plane. A typical integrand $f(k_x)$ exhibits a pair of branch points at $k_x = \pm\kappa_1$ and one or more pairs of surface wave poles. To determine the location of these poles, we first find the roots of $D^e(k_\rho) = 0$ and $D^h(k_\rho) = 0$, where $k_\rho^2 = k_x^2 + k_y^2$, and then map them into the k_x plane for each value of k_y. The correct choice of the integration paths with respect to the branch-point and pole singularities of the integrands is crucial for the success of the solution procedure [20]–[22]. It turns out that different paths must be selected for the bound regime—where the mode propagates unattenuated—and for two leaky regimes—where the mode is attenuated due to loss of energy either into the surface wave or into both the space and surface waves [7], [23]. It has been conjectured [24] that this is a possible reason why previous attempts to compute leaky microstrip modes by integral equation techniques have not been successful. More details regarding the evaluation of the integrals (5) can be found in [12] and [22].

IV. Numerical Results

In Fig. 2, we present sample dispersion characteristics of the first three higher modes in their leaky regime for a microstrip line previously analyzed by Oliner [7] using an elegant, but approximate, asymptotic approach [25]. The rigorous and asymptotic results are seen to agree quite well for the EH_1 mode; the agreement is somewhat less favorable for the higher order modes.

In Fig. 3, we present dispersion curves for the fundamental and first three higher modes in both the bound and leaky regimes for a microstrip line with a higher dielectric constant. The dashed line in Fig. 3(a) traces the largest root of $D^e(\beta) = 0$ and represents the dispersion curve of the TM_0 surface wave mode of the slab. When β crosses this line, the corresponding microstrip line mode enters the leaky regime [7]. The same structure has been previously analyzed by Lee and Bagby [26] in the bound regime only. Their results for the fundamental mode and first two higher modes are also shown in Fig. 3(a) and are seen to agree very well with our results.

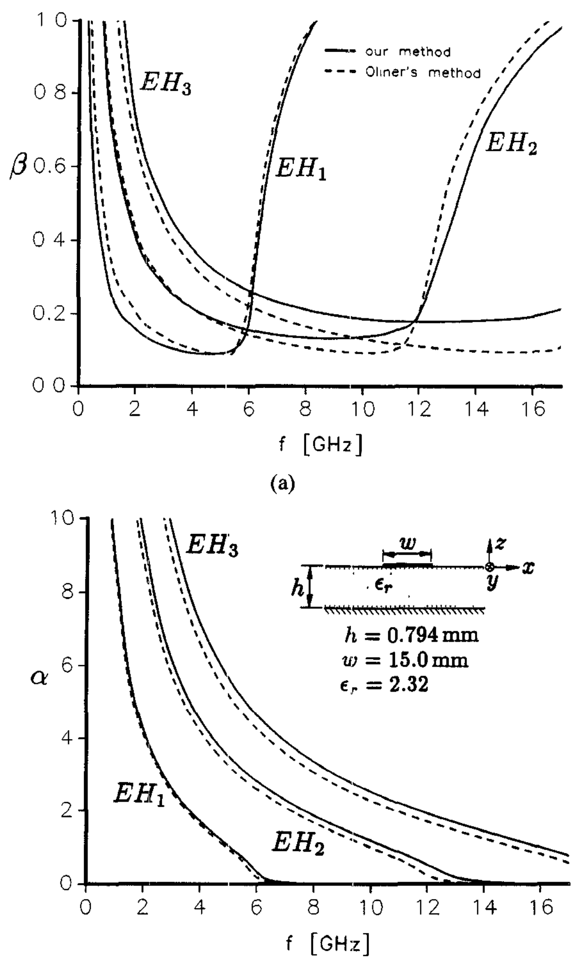

Fig. 2. Dispersion curves for the first three higher modes of an open, infinitesimally thin microstrip line. (a) Phase constants. (b) Attenuation (leakage) constants.

In Fig. 4, we present the fundamental mode dispersion curves for a microstrip line of infinitesimal thickness and for microstrips with rectangular and trapezoidal profiles. The latter may simulate the effect of the etching undercuts. We note that the dispersion curve for a microstrip of finite thickness lies below that of an infinitely thin microstrip. However, we have found that when the "equivalent width" concept is used to approximately account for the strip thickness [27], the dispersion curve actually moves up when the strip thickness is increased.

In Fig. 5, we compare current distributions for microstrips of rectangular and trapezoidal cross sections at $f = 6$ GHz. In this and subsequent figures, the current distributions are normalized, so that the longitudinal current density has a maximum magnitude of 1. Observe that, as expected, the current density is considerably higher on the bottom side of the conductor than on the top side.

In Fig. 6, we present the fundamental-mode effective dielectric constant $\epsilon_{\text{eff}} = \beta^2$ as a function of h/λ_0, where λ_0 is the free-space wavelength, for a circular-wire trans-

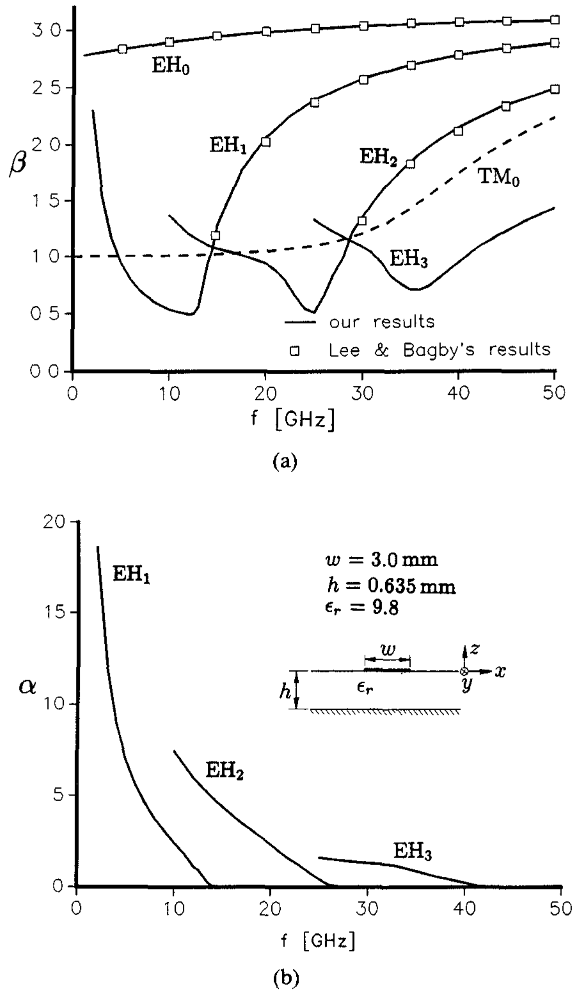

Fig. 3. Dispersion curves for the fundamental mode and the first three higher modes of an open, infinitesimally thin microstrip line. (a) Phase constants. (b) Attenuation (leakage) constants.

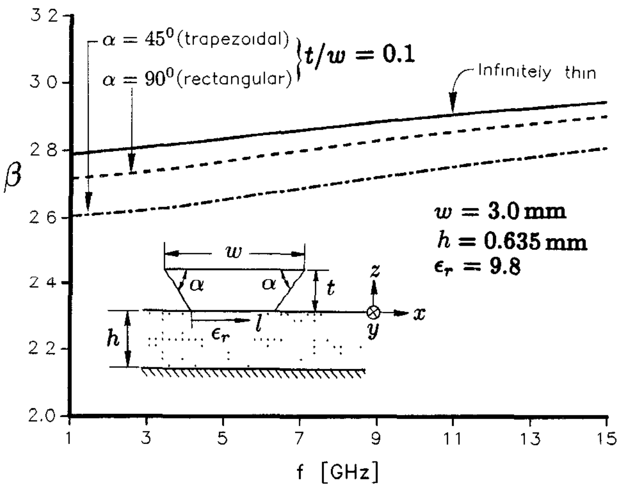

Fig. 4. Fundamental mode dispersion curves for microstrip lines of infinitesimal thickness and for microstrip lines of rectangular and trapezoidal cross sections.

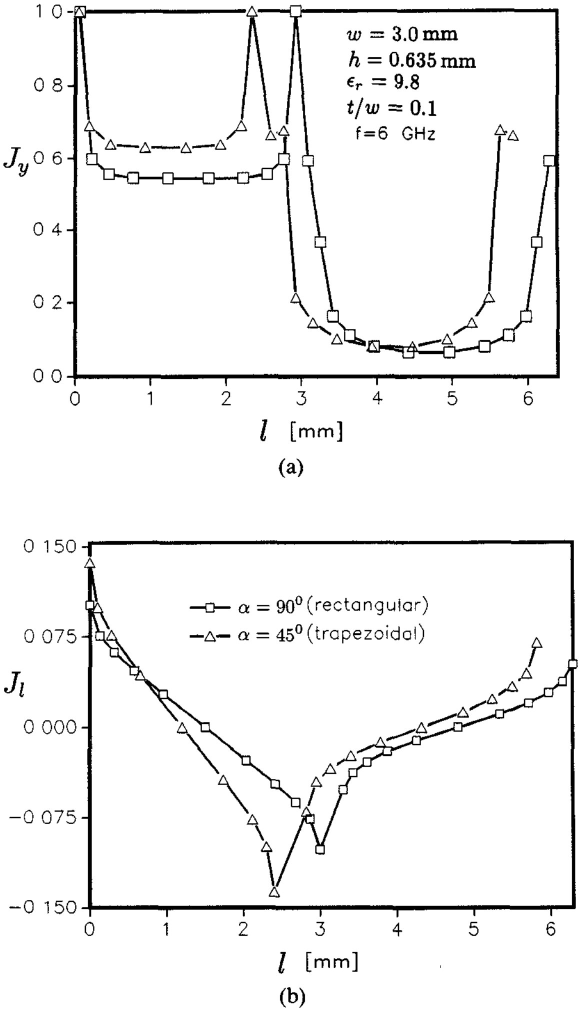

Fig. 5. (a) Longitudinal and (b) transverse current distributions of fundamental mode for transmission lines of rectangular and trapezoidal cross sections.

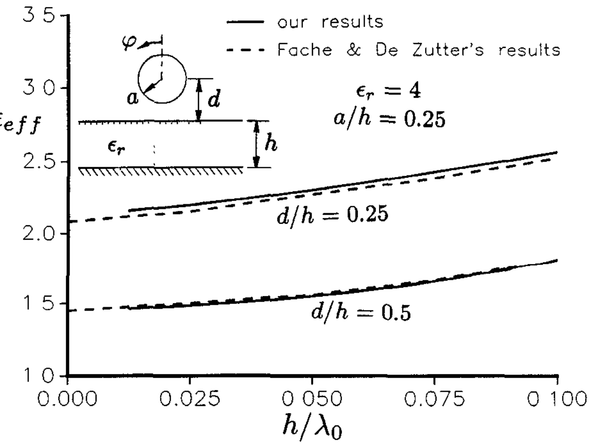

Fig. 6. Fundamental mode effective dielectric constants for a circular-wire transmission line.

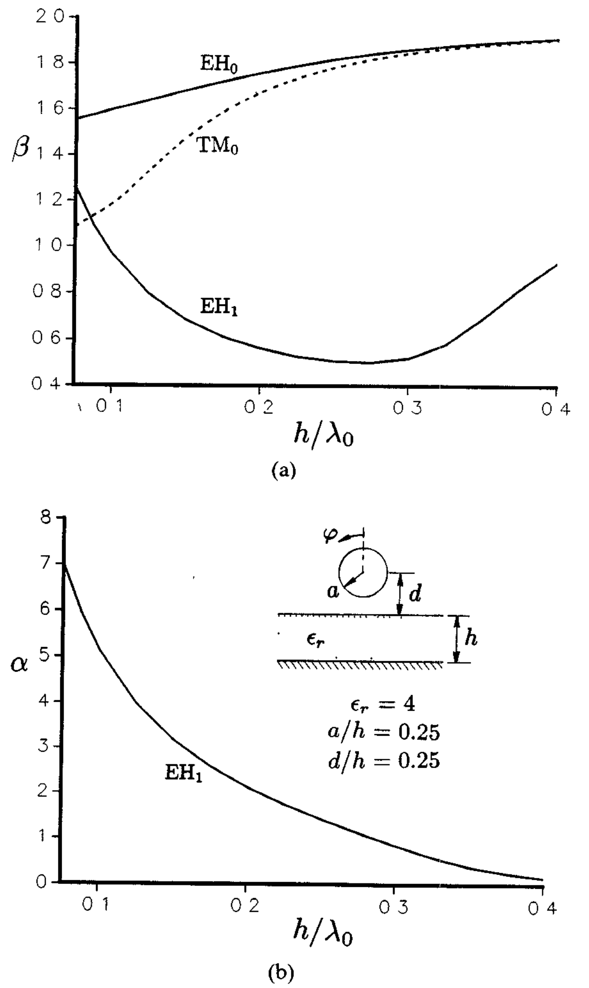

Fig. 7. Dispersion curves of the fundamental mode and the first higher mode for a circular-wire transmission line. (a) Phase constants. (b) Attenuation (leakage) constant of the higher mode.

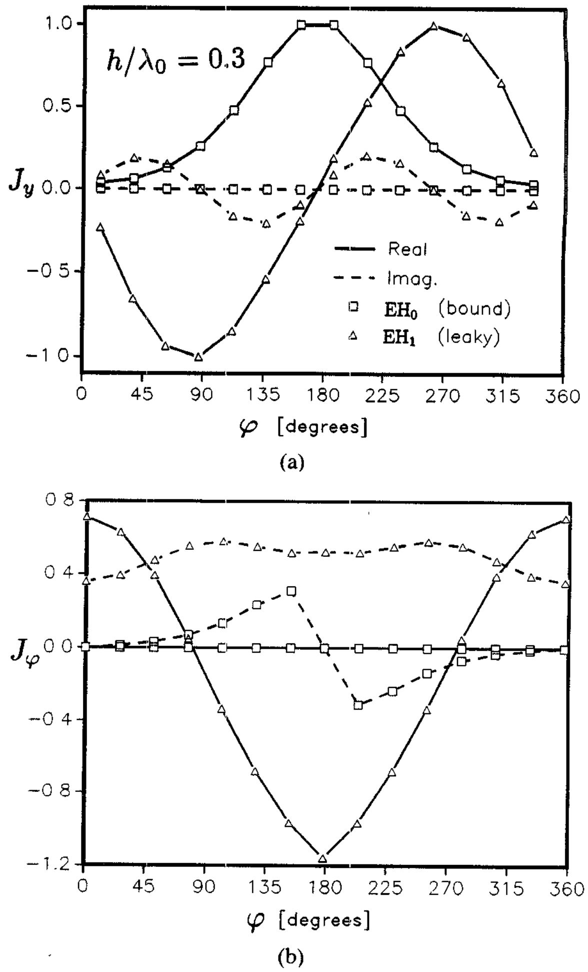

Fig. 8. (a) Longitudinal and (b) transverse current distributions of the fundamental mode and the first higher mode on a circular-wire transmission line.

mission line recently analyzed by Faché and De Zutter [6]. Their results are also plotted for comparison and are seen to agree well with ours.

In Fig. 7, we present dispersion curves of the fundamental mode (EH_0) and of the first higher mode (EH_1) for the circular-wire transmission line with $d/h = 0.25$. Observe that in the chosen frequency range the EH_1 mode is leaky throughout. In Fig. 8, we show the longitudinal and transverse current distributions for both modes at $d/\lambda_0 = 0.3$.

V. Conclusions

A rigorous solution has been presented for open microstrip transmission lines of arbitrary cross section in both the bound and leaky regimes. Dispersion curves and current distributions have been computed for microstrips of infinitesimal and finite thickness and, where possible, compared with data found in the literature.

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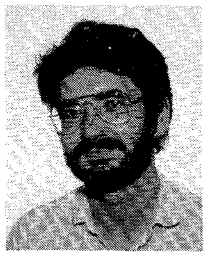

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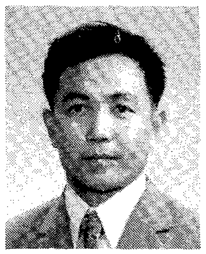

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